Math 304 (Spring 2015) - Homework 6

Problem 1.

Find the transition matrix from the basis $\{u_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\}$ to the standard basis $\{e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\}$

Problem 2.

Let $\mathbb{P}_2$ be the vector space of polynomials with degree ≤ 2. We know that $\{1, x, (x+1)^2\}$ is a basis of $\mathbb{P}_2$. Find the coordinate vector of the polynomials $p(x) = x^2 - 1$ with respect to the basis $\{1, x, (x+1)^2\}$.

Problem 3.

Given the matrix

$$A = \begin{pmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{pmatrix}$$

(a) Find a basis of the row space of A and use it to determine the rank of A.

(b) Find a basis of the column space of A.

(c) Find a basis of the null space of A.

Problem 4.

Determine whether the following mappings are linear transformations.

(a) $L : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b \\ c \end{pmatrix}$$

(b) $L : \mathbb{R}^3 \to \mathbb{R}^2$ by

$$L\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a^2+b^2 \\ c \end{pmatrix}$$

(c) Let $\mathbb{P}_3$ be the vector space of all polynomials with degree ≤ 3. The mapping $L : \mathbb{P}_3 \to \mathbb{P}_3$ by

$$L(p(x)) = p'(x)$$

where $p'(x)$ is the derivative of $p(x)$.

(d) $L : \mathbb{P}_2 \to \mathbb{P}_3$ by

$$L(p(x)) = x \cdot p(x)$$

(e) $L : \mathbb{P}_2 \to \mathbb{P}_3$ by

$$L(p(x)) = p(x) + x^2$$

Problem 5.

Find the matrix representations of the following linear transformations.

(a) Let L be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ by

$$L \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1 - x_2 - x_3 \\ 2x_2 - x_1 - x_3 \\ 2x_3 - x_1 - x_2 \end{pmatrix}$$

Find the standard matrix representation of L.

(b) The vectors

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

form a basis of $\mathbb{R}^3$. Let L be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by

$$L \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 v_1 + (x_2 + x_1) v_2 + (x_1 - x_2) v_3.$$

Find the matrix representation of L with respect to the bases $\{e_1, e_2\}$ (the standard basis of $\mathbb{R}^2$) and $\{v_1, v_2, v_3\}$.

(c) The vectors

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

form a basis of $\mathbb{R}^3$. Let L be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ defined by

$$L \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_1 v_1 + (x_2 + x_1) v_2 + (x_1 - x_2) v_3.$$

Find the matrix representation of L with respect to the standard bases.